Les Apaches de la biblioth?que infinie: Kane X. Faucher reviewed by Tom Chaffee

Contributed by Jim Chaffee

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Â The Infinite Library by Kane X. Faucher
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reviewed by Jim Chaffee Â For Shirley Hill Â

Imagine you've fallen in with bookish thugs. Hooligans of the word printed on paper. Canadian dealer in antique books Alberto Gimaldi finds himself in precisely this situation when he hires on with a collector named Castellemare who turns out to be a criminal master-librarian. Castellemare's vicious henchman Angelo first trains Gimaldi in the subtle art of retrieving escaped books only to later hunt him down to repossess books with which Gimaldi has absconded. The library from which books escape to the hands of selected readers, as did those with which Gimaldi made off, is the infinite library over which Catellemare ostensibly presides. Along the way, Gimaldi matches wits with Anton Setzer, former hireling of Castellmare, used bookstore owner and inventor-operator of an infernal device attached to the infinite library via a labyrinth. The device appends newly unwritten books to the library, a mind-boggling task given that the library holds everything written, actually or potentially, analogous to the sort of apparent paradox (only apparent) that haunts infinity in mathematical art. Â

Though the unlikely hero Gimaldi rises to the role of Camus' Sisyphus, or perhaps better to that of Mathieu Delarue on a rooftop facing the invading Germans in the final paragraphs of the final volume of Sartre's trilogy Chemins de la Liberte, it is the infinite library that is the real protagonist of the first volume of this trilogy in the works, named The Infinite Trilogy. Gimaldi is less from the street than from the stacks, but it is his love of printed words aligned on pages bound between stiff boards that heartens him to rise above his inherent poltroonery in defense of the infinite library, whether potential or actual. Â

Overhanging all is the library, purportedly infinite with the attendant conceptual problems derived of lack of imagination, incapability to conceptualize and inability to apply reason consistently with rigor that are common attributes found among the overabundance of human primates inhabiting the planet. It is expected then that characters are confused anent not only whether the library is infinite but even with respect to what it means to be infinite. Mathematicians have provided operational meaning to this notion with the simplest of constructs, but even this is beyond the ken and likely the ability to grasp of most of the human population. Witness the pathetic blunder of David Foster Wallace's attempt to explain Cantor's infinity which he himself clearly did not get and likely hadn't the intellectual chops to grasp. A review of his book Everything and More by a practicing mathematician who understands the notion of infinity, a review that not only takes on the deficiency of Wallace's understanding of infinity or much mathematics at any level but also his clumsy prose style, can be had as a free pdf file from the American Mathematical Society:
http://www.ams.org/notices/200406/rev-harris.pdf. (For an early infinite jest, see the short story Carnival by Isak Dinesen.) Â

Wallace's blunder serves as a reprimand to those who would explain infinity to others (or deny its mathematical existence), a popular literary pastime at the present: first learn the most basic of mathematical concepts outside the purely algebraic computations to which they were subjected in high school, several years late for a society that considers itself educated. (Unfortunately, algebra requires more machinery than most will be able to master to get to the work of Galois on those simple polynomials supposedly explicated in high school algebra, a topic itself more deserving of grade school). Â

The basics to which I refer is the material presented in the first eight chapters of the second edition of Walter Rudin's Principles of Mathematical Analysis, 1964 edition (a book of only about 250 pages with no prerequisite beyond elementary arithmetic, including the notion of square root). This is the most elementary of all mathematics texts, the basics of calculus as codified up to the beginning of the teentieth century. It is not a calculus text in the formula-compute presentation for engineers, but is the underpinnings of calculus without the grasp of which one cannot be said to have been educated at a college level. (The last two chapters are beyond these basics and can be better obtained elsewhere, which leaves only 180 pages to master.) The basic material presented in Rudin's elementary book is on a footing of necessity for US high school and college graduates with the broad outlines of US history, the US Constitution and the structure of the US government. It is as essential for English speaking college graduates as Shakespeare, for French speakers as Moliere, as Cervantes for Spanish or CamÃµes for Portuguese. It provides perspective on the need for the invention of rigorous infinity. Â

However, if you are born and educated in the US from the late 1980s on to the present, there is a strong likelihood, that is at least a seventy precent probability, that you are functionally illiterate. You can read most of the words but are unable to register what an author meant to say, assuming the author actually says anything at all, which is not all that common. Rudin's book is useful for testing your ability to read because he says something quite specific, operationally defined so that anyone who understands it will understand precisely what everyone else understands: i.e., what Rudin meant to say. And this understanding can be tested. Â

You can test yourself by doing all the exercises. Of course, it is this ability to objectively test understanding that makes most US students avoid mathematics. Most readers will mumble, I could read it but I am not interested, a cop out since they will not know unless they actually do read at least, say, the first chapter. The excuse of being too uninterested (or too lazy) is more likely a cover for inability, but among those functional il- to semi-literates who write, one is more likely to get "The essay doesn't go anywhere," a ruse exposed by asking where and how it gets off any path or better still, assuming an essay does follow a path to conclusion, by dragging said incompetent reader along that path. This speaks to the fungibility of incomprehension. Perhaps the problem with growing functional illiteracy in generations raised on television is that reading is active, indeed participatory, as opposed to television which is passive, or video games which are reactive. Â

The lurking of infinity as a central presence in this novel provides here an opening to describe how it is that mathematicians made the notion logically viable. This useful (though not necessary) bit can be skipped (at the reader's peril). Anyhow, it is not difficult to understand: the underlying mechanism is pre-numerical, so potentially within the grasp of those educated in the US during the last few decades. Â

Consider, then, two Texans, each with a passel of longhorns. The problem is to determine who has the most of this variety of cow (not the variant improved by Robert Blakewell of Dishley Grange in the English Midlands), but since neither is able to count past a billion they cannot quantify the elements of the herds. This does not imply they cannot compare their respective sizes. They corral the cattle into two enclosures with side-by-side exits each allowing one cow to pass through at a time, the cows exiting side by side until one corral is depleted. Without knowing how many cows either has, they have nonetheless determined who has the most. This is the essence of the notion of size in set theory. It hinges on the correspondence pairing the cows. This could also be thought of as a function between the corrals which we might as well call sets since infinity comes from set theory. Set theory was devised to place some kind of logical foundation under stuff like analysis (which includes calculus), algebra (which includes the structures needed to show that those formulae you ought to have run across allowing you to solve second degree polynomials using arithmetic operations and square roots cannot be devised for polynomials of higher degree than four), and topology which is directly set theoretic in its barest manifestation abstracting distance from analysis. Â

Many logicians believe that all of mathematics is set theory (though most mathematicians don't give a rat's ass), but read the essay, "Category Theory and the Foundations of Mathematics," British Journal for the Philosophy of Science, Volume 69, number 3, December 1986, pages 409-426 or chapter eight of Toposes and Local Set Theory, both by J. L. Bell, the latter reprinted by Dover in 2008 from the 1988 original of Oxford University Press. This provides the categorical viewpoint which is highly unappealing in terms of equipment but explicates exceedingly well the relativity of absoluteness permeating logic as rules of the game. However, note that there do exist dissidents on foundational issues who object on superstitious grounds or on the basis of inability to understand the simple concept of how to count cows. Gimaldi encounters some representatives of the latter groups on his adventure. Â

The method for determining which set has more elements is active. It specifies what must be done to verify if or not a set is bigger than another. This is an example of an operational definition; the operation can be demonstrated objectively so that there is no possible misunderstanding. In order for something to be meaningfully defined, that is, not just a lot of words strung together in proper syntax lacking semantic content, an operation is required to determine what satisfies the definition. Plenty of words have no fixed meaning, such as God or god, however one cares to consider it. The Economy as hypostatized in current nounal usage by economists and politicians and news reporters and news analysts much as medieval Scholastic philosophers reified nouns, includes one (or possibly more) of the potential usages of the word God (or god). If one listens to the context of the nominal form The Economy, no operation specifies it, particularly not the measurement of a gross domestic product, though "comparing economies" might be taken as a short hand for comparing (one of the many non-unique measures of) the gross domestic products of two nations (which cannot be said to measure any such thing as an Economy). (Note that sometimes such comparisons are made between nations and a state like California, but it is not clear how one extricates The Economy of California from that of the US, given dependencies such as the jobs program termed "Defense Spending"; would it be possible for The Economy of California to be bigger than that the of the US?) While that bogus measurement is sometimes how The Economy is used, more often it is an expression of some hulking supernatural or divine existence that must be supplicated or appeased or calmed or healed via theurgic mediation in The Market, even if the theurgy is to allow The Market to find Its own way or to produce more legal tender by some machination. This is precisely the sort of primate behavior that Thorstein Veblen would have predicted. It leads to bumfuzzled economists, policy makers, commentators and average citizens who cannot understand why, for example, after incessant entreaties to The Economy, the housing market is still not healed (mollifying The Economy ought to make housing prices rise supernaturally again) and hence to consider replacing the chief theurgist with a new intermediary and team of hierophants. Â

On the other hand, examples of terms with operational definition include square root, checkmate in chess, European call option, Champagne or Chablis (in their precise usage regarding method and origin), hydrogen, Homo sapiens, all of which can be determined by applying operations. If someone uses a term that has no such operational definition, they are speaking gibberish, making the equivalent (to us) of chimp chatter (God, the economy, evil, good, beautiful are all personal expressions that have no objective signification outside the user: dog-whistle forms of chimp chatter). Â

The operation for comparing the relative size of sets is the function, which is a relation between the two sets that specifies a unique value for each element of a given set, called the domain, in a set called the range. (Mathematicians study relations as sets, with the function playing a fundamental role; for a detailed technical discussion see Patrick Suppes, Axiomatic Set Theory, a 1972 Dover reprint of a 1960 original published by D. Van Nostrand Company). For the careless reader, the word unique might have caused confusion. Unique modifies function value for each element of the domain, meaning there is exactly one element in the range for each domain member. This does not mean that each element of the range is functionally related to an element of the domain (unique or not), nor does it mean that each element of the domain is functionally related to a different element of the range. In fact, not every element of the range need be related to an element of the domain at all. A function can take each element of the domain to the same element of the range (a constant function). Such a constant correspondence will not reveal much about the size of either set, except that the domain and range each have at least one member.   Â

By choosing one of the corrals to be the domain corral and the other to be the range, pairing each domain cow to a range cow by letting them through the gates simultaneously creates a functional relationship between the two sets of cows. This is a one-to-one function since each cow is paired with exactly one cow in the opposite corral, so long as we don't run out of cows in the range corral. If we do, then we don't have a function because we have not defined it for each cow in the domain corral. But we could swap the roles of domain and range or else define the functional value of the remaining domain cows as corresponding to the last range cow out. That function would not be one to one, but it would be onto (surjective), covering every cow in the range corral, which would show that there are more cows in the domain corral. If we had chosen the smaller passel for the domain corral, our function, though one to one (injective), would not be onto since it would not cover all the cows in the range. Hence the domain herd is smaller, since each element of the domain goes to one cow (function) and each cow in the range which has a corresponding cow in the domain has only one such originating cow (one to one). Â
In order to turn this method into counting, take our comparison set to be not cows but numbers. Identify numbers with sets by calling the number zero the set with no elements, the number one the set with the number zero as its sole element, the number two the set with only the elements zero and one, ad infinitum. The set that contains all the numbers zero, one, two, three, ... (where ... refers to the addition of one to each proceeding number, a well defined operation though eventually difficult to carry out in the real world (whatever that might be) and hence anathema to the ultra-finitists who require that to think is to do on a finite state device), is the number called Ð0 which is the smallest infinite number. (Yes, they come in various sizes; here we deal only with cardinality of the numbers, not ordinality which refers to ordering and is not particularly amusing until one encounters the infinite numbers.)
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This allows us to define a set as finite if there is some natural (counting or whole) number which, remember, is itself the set with all smaller counting numbers as members, that is the range of a one to one and onto function from said set qua number. This is an operational definition. Such a function would be to paint numbers on the cows, and of course there are several such functions since there are multiple ways to pair numbers and cows, all equivalent if numberings be consecutive. An infinite set is then one that is not finite. (Ultra-finitists would argue there cannot ever be enough paint; they would say this is the case anyway for sufficiently large numbers, which for them eventually fail to exist.) That there is such a thing as an infinite set is an article of mental gymnastics, given that to believe in the existence of mathematical objects in anything but thought is as absurd as any other superstition, be it organized or not, be it (any ilk of) Christianity or the Ghost Dance or Wicca or Kardecism or Platonism. Nor does one need be a formalist or an absolutist or some sort of constructivist or relativist to realize that mathematical truth is timeless and absolute because it is tautological (depending on the rules of deduction chosen) and implicit in the language constructs. (Consider as an example the simple proof that the square root of two is not a fraction: the logically necessary conclusion already lurks implicit in the meaning of two, square root, and fraction, all of which are defined operationally and do not tell us anything about fish, whose existence cannot be proved. But the proof does reveal that information, in the technical sense of surprise, can be gained from exploring operationally defined words in language via tautology. The same would be true of something as simple but surprising as the fundamental theorem of calculus, relating the operation of differentiation and the operation of integration, implicit in the language constructs describing said operations that anyone who understands them applies in precisely the same manner.
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Most big surprises arise when new constructs need be developed to build bridges between operational notions (the only meaningful language constructs). This would be true with Fermat's last theorem, a simple statement unproven until the relevant structures were mined and sculpted from language constructs, or the geometric structures plowed by Gauss and Riemann that led to differential manifolds and fiber bundles and so on. The process is akin to building an onion from the inside out layer by layer. Take the rigorous development of the description provided by Einstein of Brownian motion around which Wiener built his measure on an infinite dimensional function space and his homogeneous chaos. Is it any surprise that these operationally defined descriptions cross one another leading to new surprising insight (for example, the Atiyah-Singer theorem and the earlier results it generalized relating geometry to analysis which later was proven utilizing Brownian motion or the heat equation which are themselves closely related) into thought worlds precisely painted in terms describing nothing that exists anywhere. And yet there is something concrete that can be mined: one cannot, for example, expand the square root of two with whole numbers in any radix so that it repeats (or ends), so that one can never see the number that when squared would be two. This is a truth not dependent on any fact of the physical world but instead on language constructs that exist only in thought shared by all who grasp them that reaches out and slaps computer dorks, causing some to believe that there is no such thing as the square root of two hence no solution to a simple equation supposedly studied in high school in the US (and in grade school elsewhere in the "developed" world).)
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In any case, mind-boggling problems with the little infinity Ð0 bedevil the novel's characters who are associated with the infinite library. Given digestion of the background discussion, it is useful to now visit some of those characters and examine their thought defects. Woven into the action that hardens Gimaldi, this becomes a major amusement of the novel's warp and weave and reason for becoming on paper.
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Besides the original villains and whatnots Gimaldi confronts, there rise up an assemblage of odd ducks across the spectrum of behavior from academics to thugs, from purely incompetent to nearly competent, and including as well a few academic thugs. Some of them glom into secret societies resembling those which litter the penny dreadful The Da Vinci Code or its ludicrous and pretentious variant Foucalt's Pendulum. But Faucher's are not like the monotonous genre of secret societies or other vast enigmatic conspiracies found in those unintentional comedies. He instead grabs these

societies by the balls and yanks them into the world of paralibrary sciences surrounded by the reality and myth of the infinite library.

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Take Anton Setzer, former hireling of Castellemare who sets himself against the plans of his ex-employer for the library by devising a machine to force new books into the library. This is no mean feat, given that the library supposedly contains all possible books (the logical limit of a library for Platonists) so it would seem impossible to deposit a new one. Worse, Setzer's machine pushes in so many new works that old ones pop out. Now this is a difficult thing for an infinity, since there is always room for one more. I mean, take the sequence of natural numbers starting with zero and moving on one by one as one, then two, then three, ad infinitum tedium that doesn't seem to end, but nonetheless one can logically tag yet one more on the end of all those. When considered as a linear order, the set of natural numbers Ð0 is called Ï‰ to distinguish it as an ordering and not cardinality or pure size. All that has been done is to take the set Ï‰+1 = {0, 1, 2, ..., Ï‰}. No big deal. Nothing popping out here. This is the natural state of affairs with infinite sets. To see that it is the same size as Ï‰, simply take a function c from Ï‰+1 to Ï‰ by defining c(Ï‰)=0 and then defining c(n)=n+1 for all the finite n occurring prior to Ï‰. This is one to one and onto, so the two sets have the same number of elements and hence are the same size. Notice, for example, that the set of all integers can be mapped one to one and onto the set of all even integers by the function that sends an integer to its multiple by two.

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The mathematician Dedekind noted that it was possible for an infinite set to be mapped by such a one to one function onto a proper subset of itself (the even integers are a proper subset of the integers) and proposed using such property as a definition of infinity, but without something called the Axiom of Choice it is not equivalent to the definition of infinity given earlier. The Axiom of Choice is important because the acceptance of this axiom (which like Euclid's parallel axiom in Euclidean geometry, is independent of its brethren axioms in theory) changes the novel entirely. At any rate, a more interesting correspondence is that which maps the counting numbers, which march off to infinity in but one direction, one to one and onto the integers which march off to infinity in two directions (f(n)=n/2 for n even and f(n)=-(n+1)/2 for n odd, where we start with n=0). Even more interesting is the function that maps the fractions one to one and onto the integers. In the interest of not becoming excessively wearisome, we refer the reader to Suppes' book referenced above (page 173). What makes this correspondence interesting is that the fractions not only ascend forever in both the positive and negative direction but also inwardly, if you will, that is downwardly since they are densely ordered in that between any two of them resides yet another: between the fractions one-fourth and one-half is three-eighths and between one-fourth and three-eighth lie more, between three-eighth and one-half yet more ad infinitum. There are as many fractions between one-fourth and one-half as there are fractions all together. In fact, no matter how small the interval between two fractions there are as many nested within as there are fractions in totality. The counting numbers are of course discretely ordered (more precisely, well ordered) so it is amusing that the set of all of them is the same size (number of elements or cardinal number) as the set of the densely ordered fractions.

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So is there a bigger infinity than Ð0? Yes. Cantor proved that the set of all real numbers (which includes numbers like the square root two and pi neither of which belongs to the set of fractions) is a bigger infinity than the fractions (hence natural numbers). The set of all real numbers is called the continuum and is gotten by taking the collection of all subsets of integers, the so-called power set of the integers which is equivalent to the power set of natural numbers in size, or equivalently obtained by filling the holes in the fractions (one such hole is where the square root of two ought to be, as Rudin in his elementary text referenced above demonstrates), whence the name continuum since upon completing the process there remain no more holes. Due to a loophole in the notion of power set of a set (the set of all subsets of said set) stemming from a loophole in the notion of what is a subset, arises the so-called continuum problem that asks if the power set of Ð0, that is the continuum (real numbers), is the next infinity arriving after Ð0 which is named Ð1 (that there is such a lovely succession is only true for adherents of the Axiom of Choice). As it turns out, Paul Cohen famously proved this to be independent of the axioms of set theory. In essence, the power set of aleph-naught can be anything it ought to be, ought arising from certain logically necessary restrictions. This affects the novel via Setzer's machine, for one thing, but also comes into play later with the blind librarian.

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(By an enumeration of the real numbers or continuum is meant a one-to-one function from the natural numbers onto the real numbers. There is a long method of logically constructing (in thought) the real numbers (Suppes provides it in his book referenced above) that demonstrates it to be the power set of the integers, but Cantor's direct proof that no such enumeration of the real numbers or continuum can be made is so straightforward that it sometimes blows out brain cells of those who are ideologically wed to mathematical reality existing outside thought and to an absolute truth that requires finitude or at most one infinity that fits all. There are those who argue Cantor is logically wrong, but they betray mostly a misunderstanding of what it means to be a one-to-one function from the integers onto the real numbers, usually resulting from not understanding what it is to be a real number or what it is to be a function or even the nature of the game of logical derivation itself. These arguments sound like a mechanic explaining that your car is not starting because an evil spirit has taken over the whatchamacallit, or to be more specific, it's the dingflop on the skediver. This specificity is akin to a notoriously stupid example in which someone who believes pi to be represented precisely by the decimal expansion 3.1416 accuses Cantor of making the mistake of confusing radix ten representation of numbers with radix two. Suppes (section 6.7, page 189) supplies a simple and direct a construction that works for any radix greater than or equal to two that represents the real numbers in infinite expansions. But the direct proof that no enumeration of the real numbers (continuum) can exist is so simple that to not understand it belies an inability to grasp simple concepts, which is likely the underlying cause that so many cannot read. Given any explicit enumeration of real numbers by a function f(n) defined for

n=0, 1, 2, ...(as represented in any radix), it is quite simple to build a sequence in that radix which does not appear in the enumeration. Hence such an enumeration of all real numbers is impossible, and since this is the method for comparing sizes of sets, it is necessarily the case that the size of the set of real numbers is larger than that of the integers. Bonking one between the eyes is that this is independent of any radix whatsoever so long as it is at least two. It is incontrovertible and stunningly obvious. At any rate, the expansion not accounted for represents a real number not in the enumeration. With a little thought, the reader can see how to define said missing number for any enumeration (hint: in any enumeration, one may construct a new string of numbers that represents a real number not in the enumeration by picking a different digit in that radix at the nth position of the nth element f(n), which directly illustrates that the real numbers are bigger than the counting numbers or integers or fractions), but Suppes supplies a requisite number with painstaking detail for those who are not interested in doing it themselves. And though people sometimes in frustration call people who are unable to understand Cantor's proof cranks, they should instead be pitied since they are intellectually handicapped.
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Note that there exists those who do not buy the proof in pite of understanding it. Their objections generally serve a need to accept the physicality of mathematical objects or restrictions of logic to exclude certain arguments, based on a belief in some absolute truth that is not purely formal to which mathematics must adhere (and thereby throwing out most of the mathematics that has been the basis for physical science for centuries). In essence, they refuse to believe it is possible to think coherently of infinity, this denial often couched in terms of some absolute truth "out there" that transcends the game of logic. Akin to those like, say, the Austrian economics school who deny the veracity of something like Keynsian economics on ideological grounds, as opposed to those too simple minded to understand the concept of Keynes, for one common example, and who argue against a straw man. Another amusing example is the vast segment of the American citizenry who believe that once The Economy is happy The Market reflects this by rising. Consider the confused voters in the Republican primary in Florida who thought that fixing The Economy would "bring back" the housing market.)
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It might be that Setzer confounds Castellemare's plans for the library by ruining the convergence of certain sequences of books (or nets, or equivalently, filters; for simplicity's sake we assume the topology of the library is first countable). These would-be convergent sequences that satisfy the Cauchy criterion of their tail elements growing arbitrarily close and staying that way until they squeeze down (converge) to some book (or hole: a sequence of fractions can always be found that is Cauchy and converges to a number that is not a fraction, hence providing for example the square root of two which the author of this review described in another essay entitled Meaning and Almostness.) By perhaps interpolating in a sequence of constant terms, say, or some other sequence not near the book to which the sequence of library books once converged it could be that Setzer is causing there to be two or more limit points and that some of them may not be possible.
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Such is clearly the case with the fractions. For example, a sequence of fractions that converges to the square root of two is a sequence of possible numbers converging to an impossible number. (The square root of two cannot be described in a finite string in any radix, since it has no finite (repeating or synonymously, periodic) expansion or, equivalently, no representation as a ratio of two integers, and hence is impossible for humans to ever perceive. On the other hand, fractions are possible in that they are describable with finite expansions that terminate eventually into repeating patterns and of course as ratios of integers, the two being equivalent: for a constructive proof of the equivalence in radix ten translatable to any radix see Ivan Niven, Irrational Numbers, theorem nine of chapter one, number eleven of The Carus Mathematical Monographs published by the Mathematical Association of America)).
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It is possible to simply take all limits of sequences of rational numbers at one fell swoop (the set of all limit points of the set of rational numbers) to obtain the real numbers as a set (this is the topological preference: the rational numbers are dense in the continuum of real numbers). So it may be that Setzer implements a function that attempts to map the entire set of real numbers into the infinite library by taking limits of books in the library that converge to books not in the library, which if only countable, that is of size Ð0, forces out books to make room for the clearly bigger set. But it could also be that the library already contains the continuum of books (since all possible books may include impossible books? as the continuum contains impossible numbers like the square root of two or pi) and that instead Setzer overrides Castellemare's reliance on the Axiom of Choice or the Continuum Hypothesis or his reliance on the negation of one of those axioms in some particular form. Clearly bad things happen when the Axiom of Choice is considered false (or not used at all in classical mathematics which means calculus would be at risk, say) but also horrid things seem to happen when it is accepted. Some are head-bursting enough they are named paradoxes though they are not. The Banach-Tarski Paradox is a theorem that states it is possible to decompose a sphere of fixed radius (say one) into a finite number of disjoint pieces (five suffices) that can be rearranged without stretching any of them by only rotating and translating the pieces so as to form two spheres of the same size. Its proof requires the use of the Axiom of Choice and hence does not provide a method for actually accomplishing this feat, even mathematically. But think what could be done with spherical money (explaining why coins are disks, not spheres?) or gold balls were it constructive.
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The Axiom of Choice is mostly implicit in the novel, except for the existence of the blind librarian, reminiscent of a certain famous writer, and with whom it is almost explicit and through whom the dependence of the outcome of the novel on this axiom becomes transparent. The primary discussion is in the section An Inserted Letter in the Alphabet, which is The Infinite Library — Annals (1) by Jorge Luis Borges (?), the beginning of Chapter 24. Here the author of this particular piece makes a number of mistakes the origin of which are difficult to fathom. Said author (Borges?) has taken it upon him(or

her)self to organize said infinite library which would be an ordering or require an ordering or be equivalent to an ordering. What sort of order is not specified, but the would-be organizing librarian (who Gimaldi meets on his wanders and who is beset with enemies who would frustrate such an organizing or ordering) is consistently inconsistent in the constricted sense of logic. To wit, he considers an organization in which a certain phrase (a nonsense phrase, but of course the exact structure of the phrase is irrelevant) is input to a special device and that device returns all locations of books containing that phrase (though of course one could object that this is not an ordering so much as a retrieval based on some sort of record of all sequences of strings of symbols in all the books based upon an ordering) with respect to, one can only assume, a fixed origin. But the would-be ordering librarian despairs that the existence of an infinite number of books implies "... there would technically be an infinite number of books containing this exact phrase." Of course, that is incorrect, though there would potentially be such an infinity of occurrences. But then he despairs of the potentially infinite number of ways of choosing orders, and immediately he is into the problem of subsets. "Is it not logically impossible to catalogue an infinite space?"
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Well, no. Suppes explicitly supplies a particular well-ordering of the fractions (a well ordering is one that looks like the natural numbers in that every subset has a first element, that is, is discretely aligned, though it might go on infinitely long, unbranching marches in but one direction that upon leaping to the end of which one might find a new beginning, as with Ï‰+1), but in general it takes the Axiom of Choice to wave a magically well-ordering wand over any set without providing the actual order. (For the statement of the Axiom of Choice, see the little book by Thomas Jech, The Axiom of Choice which discusses in detail many of the consequences of said axiom as well as consequences of not using it in classical mathematics, and also discusses in detail many of the equivalences and not quite equivalences. The book is available as yet one more reprint from Dover. Bertrand Russell provided an enlightening example of the Axiom of Choice in terms of infinite collections of pairs of socks and pairs of shoes, the latter not requiring it. Here we take it in its equivalent statement that any set can be well-ordered.) It may require the Axiom of Choice to order this library since the orders in mind seem to be over all possible subsets and hence bring us to the continuum or more, perhaps, if the library is already a continuum, and these hand-wringing discussions of the ordering librarian may indicate that there is involved density in the sense of the fractions and the real numbers, that is, there may exist already an infinite extent of books not only outwardly but also inwardly.
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Hence, and this is the critical issue, the Axiom of Choice becomes a necessary component of the novel in that the novel's outcome depends on whether or not the reader assumes the Axiom of Choice.
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This is a major, no revolutionary, literary achievement. Without ever being directly addressed, the reader is brought into the novel despite him(or her)self, even if said reader is too dull to comprehend it. To those who can comprehend what transpires, by observing for example the ruse of a mistake planted in the blind librarian's vision of the library, it is seen that Faucher changes literature completely by making the reader an unwitting author. It would be as if instead of making characters participants in directing the writing of the novel, as with Mulligan Stew, Gilbert Sorrentino had made characters of the reader or better, had made actual readers of the characters, or even had made the characters THE reader. True, this is somewhat less, but is it not a step in that direction? Whether or not the reader wants to be, or even whether or not the reader realizes it, by his or her (unwitting?) choice regarding the Axiom of Choice, the reader is in the novel and determines its outcome. This is the case whether or not the reader finishes the novel: once sufficiently within its bowels the reader writes the novel. And there are an infinity of potential completions, where the size of the infinity now depends on the Continuum Hypothesis, more than there will ever be humans as potential readers. In essence, Faucher has made literature as real as mathematics and made the world outside the reader's head as unreal as mathematics.
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Of course, the way to understand the Axiom of Choice is as one of the many uses of the word God, which is that of a first principle or beginning, to block the can being kicked down the road. It is the confusion of those who do not realize that there is no logical inconsistency with infinite regress as mathematics has shown (see Bertrand Russell's comment on this in his History of Western Philosophy). The Axiom of Choice in its role as the well-ordering principle says only that every set can be provided a well-ordering, that is one in which every nonempty subset has a first element (hence a first element for the entire set). It says nothing about infinite regress in deductions. But this form of well-ordering gives particular comfort to the superstitious of the religious ilk, be they the heads of the organization that attacked Galileo for his monstrous telescope, those who were upset over the intrusion of the profit motive into the business of forgiveness of sins, or those who believe in golden plates now in the hands of a supernatural being. These all require a first principle and hence belief in God of the Axiom of Choice.
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In a significant moment, after being stalked, harassed and attacked by an inept group of (what else?) academics attempting to be a secret society, Gimaldi meets a true cabal, the Devorants who turn out to be as feckless as their incompetent clone. They believe Gimaldi to be a sort of savior for them in the task their faction has pursued for generations. They inhabit the library and deny it is infinite (and perhaps even deny, in coded language, its very existence) with ridiculous arguments. For example, one of them says, "No... The library is not infinite. It is... like all things... bounded. Look, here the shelf ends... And this one. How... can something infinite be composed... of finite parts?" They turn out to be a group of radical ultra-finitists making an ultra-absurd argument; clearly the set of all integers is composed of only finite parts and yet is still an infinity. So that is a specious argument. And also clear is that bounded has nothing to do with infinity, since the fractions between zero and one are clearly bounded and there are a countable infinity of them in that bounded interval. Hell, even something of infinite extent like the real number line can be bounded:

simply add a single point to that set and perform the one-point compactification with the topology to get a bounded set that looks exactly like the unit circle, bounded but still infinite. The same can be done with a single point added to the infinitely extending plane, three dimensional space or Euclidean space of any (finite) dimension (in fact, this can be related to the study of projective space in visual art dating from the Italian Renaissance and now applied in computer graphics). It seems clear that an uncountable set like the continuum will of necessity be composed of more than finite elements: it requires uncountably many expansions and "most" of those will be countably infinite without any patterns in their digits (the essence of Cantor's proof of the continuum's uncountability).
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At any rate, the ultra-finitists are destined to become inverse artificial intelligencers. The nub is, they realize that machines cannot replicate human thought and so demand that human thought be limited to what a computer can imagine. These may indeed be the same hooligans who contend that to read a book destroys its purity and so they must destroy books to save them. It is clear that Gimaldi, upon refusing to join them, wishes he has asked: If the integers are finite, what is the first integer that does not exist? The ultra-finitists demand their God be finite as well as well-ordering.
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The problem for Faucher is the disposition of this work. Given the fact that no act of innovation in contemporary literature (ironically, most especially in noncommercial literature) goes unpunished, it seems it will not be pleasant. Indeed, he will be fortunate to escape with nothing more than his work being ignored. But more likely is some form of mental anguish involving the pursuit of additional questions, such as: Is it possible to replace the Axiom of Choice here with the weaker Hahn-Banach Theorem? Or if not something quite that weak, could it be that the reader assuming the Prime Ideal Theorem, which is weaker than the Axiom of Choice and stronger than the Hahn-Banach Theorem, would suffice?
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This opens a new can of worms in mathematical literature, given that the Prime Ideal Theorem, while not implying that every set can be well-ordered since it is strictly weaker than the Axiom of Choice, does imply that every set can be linearly ordered together with the strictly stronger statement that every partial order can be extended to a linear order. This means in particular that the Prime Ideal Theorem is strictly stronger than the statement that every set can be linearly ordered, which is too weak to prove that every partial order can be extended to a linear order, and so one can ask if the line of being sucked in stops there. And so on, with the tie to the Hahn-Banach Theorem and the existence of measures on Boolean algebras and the fact that the Hahn-Banach Theorem implies the Banach-Tarski Paradox. So where does it all end? This could tie down literature professors for decades, though a direct attack on the power of the statements might fall to a frontal assault via the countable Axiom of Choice. Graduate students in mathematical literature will be busier than beavers in a Bang Brothers film (watch for work from Maurice Stoker and his students) or graduate students in economics attempting to find a universal cure for The Economy (would such be achievable through a cure for The Universe's Economy?), though that latter bit might be deemed blasphemy by the religious right. One would hope for the more controlled deadly downfall to which Jurgen brought Cabell. (Was that really it?)
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Despite the bad end we predict Faucher likely will meet, which might in fact keep him from completing the trilogy from which who knows what more literary innovations could come, his prose and insight are magnificent. Here are some samples.
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"This was not a minimalist era; more meant more. The scramble for some special and sacred identity in a hyperbolic yet homogeneous culture conveyed a different species of alienation — an alienation which had yet to be developed yearning for some kind of extreme solution. To stab and thrust desperately beyond the thick and sturdy plastic of it all..."
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"The club acted as the buffer between long bouts of waiting, a reprieve from responsibility. And this reprieve was granted by the condensation of fast-paced events into one packaged and unbreakable succession of highs, a chain of quasi-orgasmic moments of excitements that grew like bubbles and quickly aborted themselves. The bodies, the music, the fluidity of movement and sound — these were the unparalleled successes of a commercial milieu that never ceased screaming in ecstasy as if in defiance against respectability. If this place could speak of any kind of metaphysical truth, it could only speak it in the present tense, a kind of cosmically ordained episode of vomiting, taking place in a postcard art-deco Hades."
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"Books and life are imbricated, a great braid of destinies, an intertwining of purpose. Do the books live for us, or do we live for the books? I would not demote books as merely being useful for us... No, I think the purpose of human life is to produce and advance the lives of books. We are but the agents and servants of books."
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Note how the author captures the current mood of the US, especially the mood of those so-called "conservatives" (when did this word become a synonym for oafish blockheadedness? well illustrated by the false dichotomy of liberal-conservative) and especially the Tea Party who are rewriting history and the US Constitution to suit themselves and match their fantasy life (though Texas is also changing its history texts to the same end):
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"I look out on all of this and feel the swelling desire for great change. I see it in the murderous flash in a young man's eyes, I feel their need to hate like the buzzing of radio waves. Equality, tolerance, democracy, all these polite prescriptions upon them... that they begrudgingly obey for lack of any other viable alternative. The people secretly await the despot, the great era of the tyrant who will give them the permission and encouragement to exercise what they truly wish to express. Peace has been bad for all of us, a stale-making thing. It has been too long in this bloodless state, and I

feel the painful yearning of the people to return to an age of atrocities to redefine the good and the evil. Enough of the aborted apocalyptic visions that failed to materialize, and enough of the diplomatic souls that would smooth over every tension by forestalling the inevitable angry violence of the people. The people want the cruel tyrant to tell them what to do, to tell them what they feel is okay."
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Chapter 42 calls into question the very notion of order in the universe by considering the fact of order that allows physics to predict planetary orbits, for example, as disorder of a kind confounding order and purpose which pairing is a common superstitious bit of nonsense found in religious thought. It wanders on to discuss the very notion of source and light and even, yes, the Axiom of Choice and information with an aside regarding the homotopy group of the circle couched in beginnings and ends, all leading to a specious speculation regarding that vaunted need of the religious for a beginning, for a god of the well-ordering principle. Mathematics has shown that there is no logical difficulty with infinite regress, laying bare the real nature of human primate thought processes as giving rise to superstition like God or The Economy (which shows when It is happy via The Market and shows who is in Its favor by showering upon them wealth). But this discussion takes us too far afield here, though it would be useful for the reader to peruse David Ruelle's Chaotic Evolution and Strange Attractors simultaneously with the reading of this novel. And ponder the high information expression jjjyreoop as chosen by the potential Borges as infinite librarian and anti-Castellemarian, assuming they are after all different characters.